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ABSTRACT

This research examines students' ability to work with algebraic variables and their notations as a result of the type of instruction they received, and places their work on scales that illustrate its location on the continuum from arithmetic thinking to algebraic reasoning. It presents data from pre- and post-instruction clinical interviews administered to a sample of middle school students experiencing their first exposure to formal pre-algebra. Roughly half of the sample (N=15) was taught with an experimental curriculum emphasizing multiple representation skills while a comparable group (N=12) of students received traditional instruction. Analysis of the pre- and post-interviews indicate that students receiving a multiple representations curriculum are significantly more likely to show signs of algebraic reasoning than their traditionally taught peers when integrating variables into the equations they write. Additionally, they are also more likely to be able to act on a variable that is presented to them as part of a graphic representation. Contains 26 references. (Author)

Running head: Conceptual change in pre-algebra

Using Multiple Representations for Conceptual Change in Pre-algebra: A Comparison of Variable Usage with Graphic and Text Based Problems.

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Abstract

This research examines students' ability to work with algebraic variables and their notations, as a result of the type of instruction they received, and places their work on scales that illustrate its location on the continuum from arithmetic thinking to algebraic reasoning. It presents data from pre and post instruction clinical interviews administered to a sample of middle school students experiencing their first exposure to formal pre-algebra. Roughly half of the sample (n=15) was taught with an experimental curriculum emphasizing multiple representation skills, while a comparable group (n=12) of students received traditional instruction. Analysis of the pre and post interviews indicate that students receiving a multiple representations curriculum are significantly more likely to show signs of algebraic reasoning than their traditionally taught peers when integrating variables into the equations they write. Additionally they are also more likely to be able to act on a variable that is presented to them as part of a graphic representation.

Using Multiple Representations for Conceptual Change in Pre-algebra: A Comparison of Variable Usage with Graphic and Text Based Problems.

Students entering pre-algebra classes often have difficulty conceptualizing and working with variables and their notations. These problems are often exhibited as translation errors (Kieran & Chalouh, 1993) or difficulties in understanding what it is that variables represent (Booth, 1989). It has been argued that conceptualizing variables is one of the key steps in the transition from arithmetic to algebraic reasoning (Schoenfeld, & Arcavi, 1988; Lochhead, & Mestre, 1988). While it has been shown that beginning students have difficulty acting on a variable, similar errors have also been observed at the high school (Matz, 1982) and college level (Bernardo & Okagaki, 1994) when students are asked to model a word problem with an equation. For example 37 percent of college engineering students could not provide a correct equation for the relational statement "There are six times as many students as professors at this university." (Clement, Lochhead, & Monk, 1981). This suggests that the inability to act on a variable is more than a simple naive error, and that it may reflect a an integral part of the conceptual framework that a student constructs to reason algebraically. We feel that difficulties such as these derive in part from mistaken generalizations that students make when the curriculum presents algebra as a computational tool, akin to slightly more abstract arithmetic, and does not provide opportunities for conceptual change.

While in mathematical terms algebra represents a formal extension of arithmetic, many students encounter difficulty making the transition to algebra (Kieran, 1989; Herscovics, 1989). Analyses of American mathematics textbooks, reveal that much of the curriculum tends to focus on students' solving procedures (Mayer, Tajika, & Sims, 1995) This emphasis on computational skills over representation skills has the potential to increase the possibility that students will try to carry out procedures by rote, missing key opportunities to use variables. Still more worrisome is the possibility that it may reinforce arithmetic procedures that offer accurate answers, but make conceptual shift more difficult (Tall, 1989; Freudenthal, 1983).

This study examines students' problem solving with algebraic notation as a means of documenting their conceptual shift to algebraic thinking. First we present the theoretical rationale for our work in three sections: 1) a framework that describes what we mean by the term conceptual change 2) a theoretical basis for a multiple representations curriculum 3) a model of problem solving which details the roles of representing skills in problem solving. We then provide information about the 20 day experimental curriculum that was created for the intervention. The data for our analysis come from individual pre and post test clinical interviews that were given to an experimental group that received our unit, and a control group that received traditional instruction. Our analysis compares the pretest to posttest changes of the children in the two conditions working two word problems and two graphic based problems.

A framework for conceptual change

In the context of this research we have framed the conceptual shift from arithmetic to algebra as one of "building experiential bridges to a new conception" (Driver, 1988, p.144). Our perspective is similar to work that has been done trying to remedy students' scientific misconceptions by placing students in a problem based situations and asking them to justify their reasoning (Brown & Clement, 1987). The multiple representations curriculum places students in contextualized problem solving situations that require the use of tables, graphs, variables, or functions. Students work cooperatively discussing the situation and the underlying mathematical relationships, as well as translating between different types of representations. We feel that students should build on their real world knowledge and use their own sense making skills to build links to the algebraic notations and concepts that are presented in school mathematics classes. Without these connections, students can be left trying to apply a limited array of rigid arithmetic procedures where more flexible algebraic ones are needed.

It has been suggested that the ability to "act on a variable" is one of the key concepts that pre-algebra students face (Booth, 1989; Herscovics, & Linchevski, 1994). This notion is central to this research, because it links the students conceptual knowledge of algebra to their problem solving behavior with notations in a real world context. However it remains an open question as to what role instruction can play in facilitating students' ability to act on a variable, and to integrate variables into their use of mathematical notation. The research presented here details a portion of a project designed to implement a multiple representations curriculum with a group of middle school students trying make the conceptual shift to algebra, and compares their resulting patterns of variable usage with those who received traditional instruction .

The Case for a Multiple Representations Curriculum

Proponents of reform in mathematics education have argued that traditional instruction with its emphasis on computational skills does not adequately address representation skills, which may provide opportunities for students to develop key mathematical understandings. Current reform documents in mathematics education have proposed that a curriculum based on multiple representations could provide a remedy to this dilemma (California State Department of Education, 1992). National standards (NCTM, 1989) assert that by encouraging students to incorporate many different types of representations into their own sense making process, students will become more capable mathematics problem solvers. Our goal with this research was to bring classroom based research to this debate on the ways in which curriculum influences students' abilities to integrate variables, both when representations are provided as part of the problem and when they are not.

Problem Solving Framework

Recent research in cognitive science has highlighted the important role of problem representation in mathematical problem solving (Charles, & Silver, 1988). Cognitive models of problem solving have also been able to parse the process of solving a word problem into several cognitive phases, detailing the roles of each phase (Mayer, 1989): problem representation-- the solver makes a mental representation of the problem information; solution planning--the solver chooses the arithmetic and/or algebraic steps needed to solve the problem; solution execution-- the solver carries out each required arithmetic and/or algebraic step; solution monitoring-- the solver monitors his/her computations to catch errors and proceed through the steps to completion. Reducing focus on the representing phase would appear to encourage the use of rote procedures and skills. It is not surprising then that students' computation skills often exceed their understanding skills, such as interpreting a graph or the text of a word problem (Dossy, Mullis, Lindquist, & Chambers,1988).

Lab based research has supported this contention that representation skills are an important and often missing link in problem solving by training college students to represent variables on a number line and documenting subsequent increases their performance at solving two step equations (Lewis, 1989). Classroom based research has provided similar evidence, showing that a multiple representations curriculum can produce differences in the concepts students develop of variable notations (Moseley, & Brenner, 1995), and decreases in the amount of direct translation errors that students make (Brenner, & Moseley, 1993). We feel that students need to learn to understand the structure of the mathematical relationships that they will encounter in algebra classes, and that students should show this understanding through creating and working with meaningful representations. To do this however, students must be able to operate on a variable.

Definition of a variable

In order to understand students' private conception of a variable, we focused on students' ability to generate algebraic notation to represent the sturcture of two problem types. In the context of this study we have defined the understanding of a variable in terms of two criteria. First, we have defined understanding as being able to successfully write an equation that models the mathematical relation presented in the problem. For example, students can demonstrate understanding a variable by writing an expression to describe characteristics of a geometric figure's area or perimeter. Secondly, we have focused on the extent to which solvers act on variables either presented in a problem representation, or created from their mental representation of a problem that does not explicitly provide variables. This illustrates the extent to which students feel it is possible to do arithmetical operations with a variable that is presented to them. These criteria taken together give us an indication as to whether students are using new algebraic methods to approach the problem or merely attempting to apply the arithmetic skills that they have already learned.

The Experimental curriculum

Our curriculum emphasizes a problem solving process in which problem representation is the key to successful problem solving. Thus students are asked to use multiple representations to solve problems. In the context of our curriculum they see algebraic expressions and equations always in conjunction with other representations. The curriculum was implemented as a 20 day unit designed around the theme of students choosing a pizza provider for their school. It was in this context that students were asked to problem solve using the new algebraic notation. For example the second lesson of the unit focused around a situation where a computer of one of the pizzeria's suppliers appeared to be malfunctioning. Students were given tables that detailed the amounts that had been ordered from the supplier and another that detailed what the restaurant had actually received in the mail. The students were asked to compare the two tables to find the errors, and then express them in words and algebraic notation. Each ingredient listed on the table had a specific error pattern (four times the amount of olive oil ordered was received) that could be represented with algebraic notation (4x). These same expressions were revisited as part of a later lesson on graphing, in which each group of students plotted the expressions they discovered and took part in a class discussion comparing the slopes and intercepts of the lines. Overall the students worked at representing the same data across four different representations: words, tables, expressions, and graphs. Subsequent lessons had the students working with tables and graphs to determine the profitability of the pizzeria, comparing prices and areas of pizzas to determine a best buy, and writing equations to design pizzas of specific caloric content. In each of these lessons the use of representations was stressed, as was student discussion in cooperative groups.

The control classes in this study were taught with traditional direct instruction methods from a textbook which emphasized equation solving and solving word problems (Dolciani, Sorgenfrey, Graham, 1988). In the algebra chapter covered by the control classes while the treatment classes received the experimental unit, about half the textbook pages were devoted to teaching students to solve equations by transforming each side of the equation by the same operation. The students were then asked to solve word problems based around similar equations, to apply the skils that they had learned.

Overall, students in the two groups were exposed to similar mathematical content. While the experimental group was receiving the multiple representations unit, the control classes covered chapters on geometry and graphing, but unconnected to algebra. Both the experimental and control groups solved word problems, however the control group students did work more problems overall, as students in the experimental group were encouraged to problem solve within the context of the multiple representations unit. Thus, by the time of the posttest, students in both conditions had received instruction on the content targeted by our analysis.

Hypotheses

Our hypotheses stemmed from the notion that providing students with a multiple representations curriculum would promote a conceptual shift to thinking algebraically. We therefore predicted that students receiving the experimental curriculum would significantly increase, as measured by the equation variable usage scale, from pretest to posttest in their ability to model a word problem requiring a one step equation and a second situation requiring a more challenging two step equation, whereas the control class would not be expected to increase significantly on these items. Additionally, we examined differences in the procedures used by the two groups to approach the word problem tasks, and predicted that students in the control condition would be significantly more likely to attempt to solve for the variable on both word problems, showing a focus on answers (Booth, 1989) that has been linked to arithmetic thinking.

We also predicted that students in the experimental condition would post significant pretest to posttest increases on the graphic variable usage scale, for both the perimeter and area problems due an increased ability to act on a variable. In contrast, we predicted that the traditionally taught students would not to increase significantly on these items.

Method

Participants

Four intact classes at three junior high schools in a small urban area in Southern California were selected to receive the multiple representations unit. Four students were selected to participate from each of these classes, but one student dropped out of the study for a total of 15 experimental students. Students in the control condition were also sampled in groups of four from the three traditional classes for a total of 12 control students. Students were nominated to participate in the study by their teacher who was asked to provide groups of four students from each class consisting of Latino male and female students, as well as an Anglo male and female students at varying levels of mathematical achievement. This yielded a total of 27 respondents balanced for gender, ethnicity, and prior achievement. All students were all in their first year of pre-algebra.

To minimize teacher effects, three of the four teachers participating in the study taught both a treatment class that received the multiple representations unit and a control class. The control class used a textbook (Dolciani, Sorgenfrey, Graham, 1988) that was the regular text at all three schools.

Materials

Students were presented with the problems in 9 sections with each problem clearly displayed on 8 1/2" x 11" sheet of white paper. The students were told that they could write on the sheets during the course of the interview to show their work. From the 9 sections that also included items on interpreting graphs, and tables, four sections that focused on notation were chosen for the analysis. Two items focusing on graphic representations and two text based problems were selected on the basis that they showed students' ability to use notation and act on a variable with and without the aid of a representation. The word problems consisted of a one step and a more challenging two step problem (see Table 1). Each was constructed to provide a reasonable real world context that the students could reason with as part of their efforts to represent the underlying mathematical relation.

The graphic problems were based around applying the geometric formulas for area and perimeter to the variables presented in a diagram (see Figure 1). The perimeter problem asked students to provide an algebraic expression that would characterize the perimeter of the irregular shape presented to them with variable notations at its edges. The area problem prompted students to fill in a table they were given to represent the sub-sections of a diagram as well as the area for the entire diagram[1]. The table format encouraged students to consider the area of each section and their contribution to the total area. All interviews were audio taped.

Procedure

All students were interviewed individually, directly before and after they received either five weeks of experimental or control curriculum. The interview process typically took 35 to 45 minutes to complete. The think aloud format was explained to all the participants, who were instructed to reveal their thoughts as completely as possible during the course of the interview. The interviews were conducted by three members of the research team, working off a sheet of common probes to insure comparability between respondents.

All of the items were placed in front of the student one at a time. Each student was presented with two word problems and asked to write equations that would represent each of the problems. The think aloud procedure was repeated to them. Later in the interview students were given a problem that asked them to represent the perimeter of an irregular shape with an algebraic expression, and a final problem that asked them to fill in a table to partition the area of a diagram.

Analysis

All pretest/post interviews for both conditions were then coded by the authors using sets of specific questions and criteria, designed to highlight conceptual differences between the two conditions. All written work produced by the student for the word and graphic problem sections was recorded and scored with its respective scale.

Both the equation and graphic variable scales were created to address issues that have been raised as central to the shift from arithmetic to algebraic reasoning such as using the notations and conventions of algebra (Kieran, 1989), acting on a variable (Booth, 1989), and their use of algebraic structure (Matz, 1982). The highest level of each scale indicates the presence of correct algebraic structure and a variable that is acted upon by an operation.

A seven point scale was constructed for the word problems to capture levels of variable usage. The scale ordinates different types of conceptual understanding items to illustrate the differences between a student who was too confused to respond, and one that was able to act on the variable to the extent that he/she produced an accurate equation with an integrated variable (see Table 2). The high scores on the scale, 6 and 7, indicate that the student was able to write an equation with appropriate use of variables. A score of 5 was assigned to equations that used a variable but did not use correct algebraic structure. A 4 was given to all equations that displayed correct arithmetic but did not include a variable. A 3 was assigned to all incorrect number sentences, vertical arithmetic was given a 2, and a 1 was assigned to students who could not produce coherent notation. Thus, while scores from 5 to 7 represent levels of reasoning with an algebraic variable, scores from 2 to 4 represent levels of arithmetic focus.

All equations written by the students were scored by a researcher who did not know which students were from experimental and which were from control conditions. Paired t-tests were performed comparing the pretest to posttest increases on the equation variable usage scale for each condition separately. Due to the fact that it was expected both groups would score higher on the posttest, one tailed tests of significance were used. To control for inflation of alpha, which is inherent in multiple t-tests, a Bonferroni correction was used, yielding a one tailed test with alpha=.05.

An analysis of the coded audio tapes was conducted to examine differences in the word problem solving procedures of the two conditions. These measures investigated the possibility that traditionally taught students might exhibit a "focus on answers" (Booth, 1989) significantly more often than those in the experimental condition. This was scored dichotomously by grouping the number of students who did not solve for the variable or who did so after they had completed their equation and comparing them with those who calculated a numerical value as a step to writing an equation. This resulted in separate 2x2 tables for both the one and two step problem, tested for significance with Pearson's Chi square.

For the graphic variable problems we again conducted a blind qualitative comparison of the written work produced by students in the two conditions for the area and perimeter problems. A 5 point scale was then constructed for these items and they were coded with a 1 standing for a student who gave an incoherent answer or simply none at all, to a 5 for a student who was able to produce an accurate expression (see table 3).

Again paired t-tests were performed comparing the pretest to posttest increases on the geometric variable usage scale for each condition separately. A Bonferroni correction was also used, yielding a one tailed test with alpha= .05.

Results

Word problems

Students in the experimental condition showed substantial differences from pretest to posttest (see Table 4) as they produced significant difference on both the one step problem $t(14)= 5.13$, 1-tailed, $p=.000$* and the more challenging two step problem $t(14)= 3.42$, 1-tailed, $p=004$*. The results were most pronounced for the one step problem which revealed that students in the experimental condition produced a mean of 5.13 which was indicative of their including variables in their mathematical notation.

The control group showed minimal improvement on the text based problems, despite their more extensive practice with solving word problems. In addition the control class displayed more of an arithmetic focus at posttest than did the experimental group on the one step problem $\chi^2(1, N=27)=4.63$ $p<.05$* (see Table 5) as well as the two step problem $\chi^2(1, N=27)=3.84$ $p<.05$* (see Table 6).

Graphic based problems

Results for the graphic problems were somewhat mixed (see Table 7). The experimental condition produced significant gains on both the perimeter problem $t(14)=4.96$, one tailed, $p=.000$* and the area problem $t(14)= 2.44$, one tailed, $p=.028$*. Additionally, by the posttest, 13 of the students used variables appropriately for the perimeter problem and 9 used them appropriately for the area problem.

Control students showed significant gains from pretest to post test on the perimeter problem $t(11)=2.60$, one tailed, $p=.025$*, but did not with the area problem $t(11)=1.08$, one tailed, $p=.305$ ns. However, only half of the students in the control condition reached levels 4 or 5 on the graphic variable usage scale for the perimeter problem, the levels that indicate that students used variables appropriately, as compared to 87% of the treatment group.

Discussion

An important part of the transition from arithmetic to algebra is a student's ability to incorporate a variable into mathematical reasoning that previously focused on much more explicit operations and values. The control class with its focus on solving for the variable and tendency to represent mathematical relationships with explicit values exemplified an arithmetic perspective in their mathematical reasoning. Consequently they appeared to have difficulty moving beyond explicit values and procedures and into variables. While these same procedures may produce short term results in prealgebra, it is difficult to see how these skills will assist them in reaching the long term goal of mastering the concepts of algebra, and in fact they may hinder transition to algebra as others have suggested (Booth, 1989; Tall, 1989).

The finding that students receiving a multiple representations curriculum were more able to act on variables as well as more likely to introduce them into their written equations, suggests that students in the two conditions are working from different conceptual perspectives. Students in the control classes tended to rely on explicit values and operations that are common to arithmetic problems. Control students were also less able to reason about where to place a variable within the mathematical structure that was presented in the problem than their experimentally taught peers. Thus students in the experimental condition were able to perform operations on variables and include them as part of their sense making skills. This provides an early indication that the conceptual knowledge of algebra and its notations could be linked to their real world knowledge of problem solving through enhanced representation skills. Despite the high levels of practice in arithmetic experienced by the control students, they did not link their arithmetic knowledge to the problem contexts as often as those in the experimental condition. Thus for students receiving the multiple representations unit, their knowledge of arithmetic was integrated with the new concept of a variable to a much greater extent then those who received traditional methods, due to their ability to represent a situation with algebraic notation.

These findings are similar to our other work which has shown that a multiple representations curriculum can influence not only the type and frequency of representations used, but also significantly improve students' performance at solving a function based word problem (Brenner et al., in press). The findings presented here are also consistent with lab based research in cognitive science, that has highlighted the importance of having a conceptual grasp of what a variable represents in its context, as one of the key cognitive aspects of word problem solving. (Hegarty, Mayer, & Monk, 1995).

Conclusions

Overall, these findings extend our earlier work detailing the cognitive out comes that can result from a multiple representations curriculum. Further, they lend support to the notion that a multiple representations curriculum can make substantial differences in the ways that students conceptualize variables and their notations. Perhaps with greater implementation of the ideals of the mathematics reform movement, future research will be able to document the persistence of these findings. Also at issue is the level of preparedness that a multiple representations curriculum will provide for more advanced reasoning with variables. It remains an open question as to what long term benefits this type of experimental curriculum could provide. This research marks an attempt to bring classroom based data to the debate over how students should make the difficult transition from arithmetic to algebra. It is hoped that further research will be able to document swifter transitions to algebra that may arise from the further study and implementation of these methods.

Footnotes

[1] Although this was not specifically part of our analysis, the area problem was also an illustration of the distributive property, showing that portions of one side can be multiplied by another side individually and then added together (i.e. (ab + ac)) or summed and then multiplied by the remaining side (i.e. a(b+c)) and obtain the same result.

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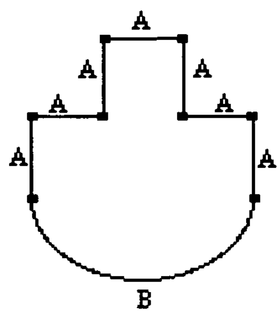
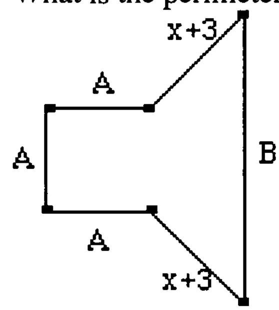
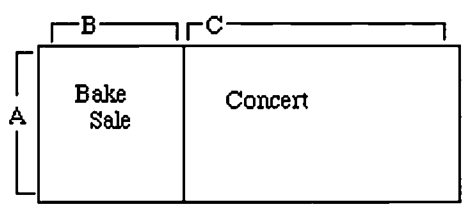
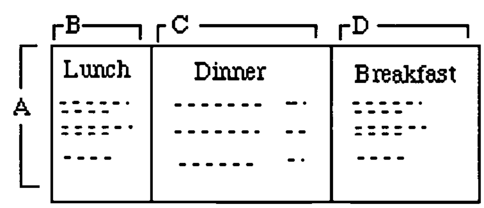
Figure 1

<u>Graphic Problem Measures for Pretest and Posttest Interviews</u>

<table>
<tr><th>Graphic Problems</th><th>Pretest</th><th>Posttest</th></tr>
<tr><td>Perimeter</td><td>What is the perimeter of this shape?

</td><td>What is the perimeter of this shape?

</td></tr>
<tr><td>Area</td><td>Arturo is planning a concert and bake sale to raise money for the track team. He needs to know how to divide up the field so that it will look like this diagram. Can you help him by filling in the table?

</td><td>Nina wants to make some new menus for her restaurant. She needs to know how to divide up the menu so that it will look like this diagram. Can you help her by filling in the table?


<table>
<tr><th></th><th>Length</th><th>Width</th><th>Area</th></tr>
<tr><td>Lunch</td><td></td><td></td><td></td></tr>
<tr><td>Dinner</td><td></td><td></td><td></td></tr>
<tr><td>Breakfast</td><td></td><td></td><td></td></tr>
<tr><td>Entire Menu</td><td></td><td></td><td></td></tr>
</table></td></tr>
</table>

Table 1

Word Problem Measures for Pretest and Posttest Interviews

Measures	Pretest	Posttest
Word Problems One-step	Lauren has a big math assignment this evening. She has already solved 21 of the problems. There are 30 problems in all for her homework assignment. How many more does she need to solve?	Jana sold twice as much candy as Raul. She sold 42 boxes of candy. How much did Raul sell?
Two-step	Francisco bought his mother a card and some candy for Valentine's day. He spent $6.00 altogether. The card cost $2 and he bought 5 ounces of candy. How much did each ounce of candy cost?	Mr Smith's class held a big book sale to raise money for the computer lab. Each book cost $2. Before the sale began, the class had $45 in the computer fund. After the book sale there was $147 in the computer fund. How many books did the class sell?

Table 2

Word Problem Variable Usage Scale

Scale Value	Definition	Example
1= none	respondent is baffled, writes nothing	blank
2=vertical arithmetic	numbers lined up in columns and rows to carry out operations	30 -20
3= partial arithmetic	arithmetic operations and numbers presented without the correct mathematical structure of a number sentence	6-2=4÷5
4=number sentence	correct values and operations with no variables	30-21=9
5=partial algebra	variables are used with numbers and operations, but lack correct mathematical structure	21-30=N
6=full algebra	variable, number, and operations are used to make a viable equation	30-21=N=9
7=integrated variable	variable is integrated into a viable equation, instead of isolated	30-X=21

Table 3

Graphic Problem Variable Usage Scale

Scale Value	Definition	Example
1= none	respondent is baffled, writes nothing	blank
2=numerical answer	student assigns numbers to sides of figure and calculates a numerical answer	6.5 inches
3= inappropriate variable usage	student uses a variable but the expression does not fit the required answer	A x 2+X=
4=accurate use of variable, syntax error	student writes an expression that answers the question but forgets parentheses or uses exponent instead of multiplying	B+C+D x A
5=accurate expression	student answers question with correct use of variable and syntax	(B+C+D) x A=

Table 4

Pretest and Posttest Results for Word Problem Variable Usage Scale by Group

	Pretest		Posttest			
Group	Mean	SD	Mean	SD	t-value	p
Experimental						
(n=15)						
One-Step	2.86	1.55	5.13	1.59	5.13	.000*
Two-Step	2.26	1.16	4.20	1.82	3.42	.004*
Control						
(n=12)						
One-Step	3.08	2.02	3.91	1.16	1.52	.157
Two-Step	3.00	1.34	3.58	1.31	1.17	.267

Table 5

Students Focusing on Answer of One-Step Problem at Posttest by Treatment Group

	Number of Students Who Displayed Focus on Answers	Number of Students Who Did not Display focus on Answers
Experimental	5	10
Control	9	3

χ^2 (1, N=27)= 4.63 $p<.05$*

Table 6

Students Focusing on Answer of Two-Step Problem at Posttest by Treatment Group

	Number of Students Who Displayed Focus on Answers	Number of Students Who Did not Display Focus on Answers
Experimental	7	8
Control	10	2

χ^2 (1, N=27)= 3.84 p<.05*

Table 7

Pretest and Posttest Results for Graphic Variable Usage Scale by Group

	Pretest		Posttest			
	Mean	SD	Mean	SD	t-value	p
Experimental						
(n=15)						
Area	2.86	1.50	3.60	1.18	2.44	.028*
Perimeter	2.60	1.36	4.46	1.24	4.96	.000*
Control						
(n=12)						
Area	2.83	1.46	3.16	1.52	1.08	.305
Perimeter	2.16	1.26	3.25	1.28	2.60	.025*

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